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Inflation conditions for non-BPS D-branes

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Abstract
We investigate the effective action for a non-BPS brane in a time-dependent embedding. This action is considered as the action for tachyon and embedding coupled to the brane gravity. We derive the slow-roll parameters from this model.

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1. Introduction

The observation of cosmic microwave background (CMB) has provided good evidence that the universe, being described by the Friedman–Robertson–Walker (FRW) metric, underwent a period of acceleration in its early times. The data from Type Ia super-novae pointed out that the universe has accelerated very recently and remains in this state up till now. To describe these phenomena in the string theory approach makes one of the most important theoretical challenges. In seeking a description of these phenomena, one may pass to an effective field theory in the low energetic approximation. The result then is a supergravity theory in ten dimensions. In order to obtain 3-spatial dimensions, one shall either construct a spontaneous compactification scenario or postulate that universe is a type of a 3-brane. One of the most popular recent approaches to the inflation problem is to use an open string tachyon on a non-BPS brane as an inflaton [1]. The non-BPS states are then realized as the bounded states of a brane–antibrane system with tachyon condensation [2]. In this approach (scenario), the inflaton potential should, in principle, be computed directly by substituting the complete superpotential into the supergravity F-potential. Then the break of supersymmetry in the brane–antibrane system leads to a subtle problem. The problem is that the exponential tachyon potential cannot produce the last 60 e-folds [3]. In the other scenario, the role of inflaton is played by the separation between D-branes [4, 5]. Both of the scenarios above are accommodated in the form of a hybrid inflation where the tachyoinc open string fluctuations end inflation [6]. In this paper, we shall study the inflation conditions in the system with a tachyon field. This system corresponds to a non-BPS brane and is described by the DBI-like action. This system is embedded in the background produced by BPS branes. The effective action for a non-BPS brane consists of the Hilbert–Einstein action and the DBI-like action. The form of the slow-roll parameters is obtained from this effective action.

2. Non-BPS Dp-branes

An N coincident BPS Dk-branes produced a background in which metric G_{MN}, a dilaton ϕ and the RR potential $\widetilde{A}_{(k+1)}$ are given by [7]

$$G_{MN}\,\mathrm{d}X^M\,\mathrm{d}X^N = \lambda\eta_{\mu\nu}\,\mathrm{d}X^\mu\,\mathrm{d}X^\nu + \lambda^{-1}(\mathrm{d}r^2 + g_{mn}\,\mathrm{d}X^m\,\mathrm{d}X^n), \tag{2.1}$$

$$\mathrm{e}^{-\phi} = \lambda^{(3-k)/2}, \tag{2.2}$$

$$\widetilde{A}_{(k+1)} = \lambda^2\,\mathrm{d}t \wedge \mathrm{d}X^1 \wedge \cdots \wedge \mathrm{d}X^k, \tag{2.3}$$

where the warp factor λ is $\lambda = [H_k(r)]^{-1/2}$ and $H_k(r)$ is a harmonic function. In the warped compactifications, the factor λ is constrained [8]. These BPS Dk-branes warp $(8-k)$-dimensional manifold Y with a metric g_{mn}.

Let us consider a non-BPS Dp-brane (with $p < k$) which is embedded in the background described above. The action for this non-BPS brane is [9]

$$S = -T_p\int \mathrm{d}^{p+1}\xi\,\widetilde{v}(T)\,\mathrm{e}^{-\phi}\sqrt{-\det(\gamma_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + B_{\mu\nu} + \partial_\mu T\,\partial_\nu T)} + T_p\int v(T)\,\mathrm{d}T \wedge X^*\widetilde{A}_{(k+1)}, \tag{2.4}$$

where T is a tachyon field with a potential $\widetilde{v}$.

For embedding in the following form

$$X^M(t,\xi^1,\ldots,\xi^p) = (t,\xi^1,\ldots,\xi^p, r(t,\xi^1,\ldots,\xi^p),\theta^1,\ldots,\theta^{8-p}), \tag{2.5}$$

the action (2.4) (for $F = B = 0$) takes on the form

$$S = -T_p\int \mathrm{d}^{p+1}\xi\,\widetilde{v}(T)\,\mathrm{e}^{-\phi}\sqrt{-\det(\lambda\eta_{\mu\nu} + \lambda^{-1}\partial_\mu r\,\partial_\nu r + \partial_\mu T\,\partial_\nu T)} + T_p\int v(T)\,\mathrm{d}T \wedge X^*\widetilde{A}_{(k+1)} \tag{2.6}$$

and the induced metric $\gamma_{\mu\nu}$ is

$$\gamma_{\mu\nu} = \lambda\eta_{\mu\nu} + \lambda^{-1}\partial_\mu r\,\partial_\nu r. \tag{2.6a}$$

The DBI-like part can be rewritten as

$$S = -\int \mathrm{d}^{p+1}\xi\,v(T)\lambda^{(4+p-k)/2}\sqrt{\det(I + \eta^{-1}S)}, \tag{2.7}$$

where the matrix S has the following entries:

$$S_{\mu\nu} = \lambda^{-2}\partial_\mu r\,\partial_\nu r + \lambda^{-1}\partial_\mu T\,\partial_\nu T, \tag{2.8}$$

and $v(T) = T_p\widetilde{v}(T)$. We restrict ourselves to the case when the tachyon T and the field r depend only on time t. Thus the action takes on the form [10]

$$S = -\int \mathrm{d}^{p+1}\xi\,v(T)\lambda^{(4+p-k)/2}\sqrt{1 - \lambda^{-2}\dot{r}^2 - \lambda^{-1}\dot{T}^2} + T_p\int v(T)\,\mathrm{d}T \wedge X^*\widetilde{A}_{(k+1)}. \tag{2.9}$$

The DBI-like action (2.9) is appropriate for distances r larger than the fundamental string length l_s between a Dp-brane and a background k-brane. Otherwise one should replace this action with the action of a complex scalar tachyon field with a potential. This potential was calculated in [11] for $p = 3$ and $k = 5$.

3. Inflation and slow-roll parameters

We investigate cosmological consequences of the action (2.9). This action is considered as the action for the fields r and T which are coupled to the Einstein gravity on the worldvolume of the brane. The action for the tachyonic field only is considered in [12]. We also restrict the dimension p of a non-BPS brane to 3. Thus the effective action for a non-BPS D3-brane is given by

$$S_{\text{eff}} = \int \mathrm{d}^4x \frac{m_P^2}{2}\sqrt{-\gamma}R + S[r, T], \tag{3.1}$$

where the four-dimensional Planck mass m_P is equal to $(8\pi G)^{-1/2}$ and the scalar curvature R is obtained from the metric (2.6*a*). The action $S[r, T]$ is given by (2.9). In the case when r is homogenous and depends on time t the induced metric on the worldvolume has the form

$$\mathrm{d}s^2 = -\sigma\,\mathrm{d}t^2 + \lambda\delta_{mn}\,\mathrm{d}x^m\,\mathrm{d}x^n, \tag{3.2}$$

where

$$\sigma = \lambda(r) - \frac{\dot{r}^2}{\lambda(r)}. \tag{3.3}$$

The Lagrangian for fields r and T is obtained from (2.9) and has the form $L = v(T)\,\mathrm{e}^{-\Phi}\sqrt{1-\dot{T}^2/\sigma}$ where $\Phi = \phi - \frac{3}{2}\ln\lambda - \frac{1}{2}\ln\sigma$. The energy–momentum tensor for the above system is

$$T_{00} = \frac{\sigma v\,\mathrm{e}^{-\phi}}{\sqrt{1-\dot{T}^2/\sigma}}, \tag{3.4}$$

$$T_{mn} = -\lambda v(T)\,\mathrm{e}^{-\phi}(1-\dot{T}^2/\sigma)^{1/2}\delta_{mn}. \tag{3.5}$$

Thus the field equations $R_{\mu\nu} - \frac{1}{2}\gamma_{\mu\nu}R = 8\pi G T_{\mu\nu}$ take on the form

$$H^2 + \frac{1}{2}\frac{\dot{\sigma}}{\sigma}H\left(\frac{\sigma}{a^2} - 1\right) = 8\pi G\frac{\sigma v\,\mathrm{e}^{-\phi}}{3\sqrt{1-\dot{T}^2/\sigma}}, \tag{3.6}$$

$$2\frac{\ddot{a}}{a} + H^2 - \frac{\dot{\sigma}}{\sigma}H = 8\pi G\sigma v\,\mathrm{e}^{-\phi}(1-\dot{T}^2/\sigma)^{1/2}, \tag{3.7}$$

where $a^2 = \lambda$ and the Hubble parameter H is given by $H = \dot{a}/a$. The equation of motion for T is obtained from the Lagrangian L:

$$\frac{\ddot{T}}{1-\dot{T}^2/\sigma} + (6-k)\,H\dot{T} + \frac{v'}{v}\sigma - \frac{1}{2}\frac{\dot{T}}{1-\dot{T}^2/\sigma}\frac{\dot{\sigma}}{\sigma} = 0. \tag{3.8}$$

For $\sigma = 1$ the field Φ is related to the warp factor λ as follows: $\mathrm{e}^{-\Phi} = \lambda^{\beta+1/2}$. Let $\beta + 1/2 = (3-k)/2$. Thus equations (3.6) and (3.7) are reduced to the form (note that $\mathrm{e}^{-\phi} = a^{2\beta+1}$)

$$H^2 = 8\pi G\frac{va^{2\beta+1}}{3\sqrt{1-\dot{T}^2}}, \tag{3.9}$$

$$\frac{\ddot{a}}{a} = 8\pi G \frac{v a^{2\beta+1}}{3\sqrt{1-\dot{T}^2}}\left(1 - 3\dot{T}^2/2\right). \tag{3.10}$$

The constraint $\sigma = 1$ says that the metric (3.2) is space flat with the scale factor a^2. For $\beta = -1/2$ (which corresponds to $k = 3$) and $\sigma = 1$, we obtain the well-known form of the equations. We shall only consider the case when $\sigma = 1$.

In order to get conditions on inflation, we use the slow-roll parameters from [13]. In [14], a similar problem was considered but it did not account for the dilaton field. These slow-roll parameters are defined as follows:

$$\varepsilon_{i+1} = \frac{\mathrm{d}\ln|\varepsilon_i|}{\mathrm{d}N}, \tag{3.11}$$

where $\varepsilon_0 = H_0/H$ and H_0 is the Hubble parameter at some chosen time. The Hubble parameter is considered here as the function of the e-foldings number N given by $N = \int_{t_{\text{init}}}^{t_{\text{end}}} H\,\mathrm{d}t$. The parameters ε_i as the functions of time t are governed by the following equation:

$$H\varepsilon_i\varepsilon_{i+1} = \dot{\varepsilon}_i. \tag{3.12}$$

The first two slow-roll parameters have the form

$$\varepsilon_1 = -\frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}N} = -(\beta+1/2) + (\beta+2)\,\dot{T}^2, \tag{3.13}$$

$$\varepsilon_2 = \frac{1}{\varepsilon_1}\frac{\mathrm{d}\varepsilon_1}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}N} = \frac{2(\beta+2)\dot{T}\ddot{T}}{[-(\beta+1/2)+(\beta+2)\dot{T}^2]H}, \tag{3.14}$$

where we used equations (3.8) (with $\sigma = 1$), (3.9) and the relation $\mathrm{d}T/\mathrm{d}N = \dot{T}/H$. Thus equation (3.9) as the function of ε_1 takes on the form

$$H^2\sqrt{1 - \frac{2}{3}\varepsilon_1} = \frac{8\pi G}{3\sqrt{3}} v a^{2\beta+1}\sqrt{2\beta+4}. \tag{3.15}$$

Differentiation of the above equation, with respect to the cosmological time t, gives (where we used (3.12))

$$-2\sqrt{(\beta+2)\widetilde{\varepsilon}_1}\frac{\left[1 - \frac{2}{3}\varepsilon_1 + \frac{1}{6}\eta\varepsilon_2\right]}{1 - 2\varepsilon_1/3} = \frac{v'}{vH}, \tag{3.16}$$

where $\widetilde{\varepsilon}_1 = \varepsilon_1 + \beta + 1/2$ and $\eta = \varepsilon_1/\widetilde{\varepsilon}_1$. The second derivative of (3.15) gives

$$\begin{aligned}(2\varepsilon_1 - \eta\varepsilon_2) &+ \frac{\varepsilon_2}{3}\left[5\varepsilon_1 - \frac{\eta(3\eta-2)}{2}\varepsilon_2 - \eta\varepsilon_3\right]\gamma^2 \\ &+ 4\widetilde{\varepsilon}_1\left(1 - \frac{2}{3}\varepsilon_1 - \frac{1}{6}\eta\varepsilon_2\right)\left(1 - \frac{2}{3}\varepsilon_1 + \frac{1}{6}\eta\varepsilon_2\right)\gamma^4 = \frac{v''}{(\beta+2)\,vH^2},\end{aligned} \tag{3.17}$$

where $\gamma^2 = (1 - 2\varepsilon_1/3)^{-1}$. Up to the first order in ε_1 and ε_2, we get

$$\varepsilon_1 = \left(\frac{3}{2}\right)^{5/2}\frac{m_{Pl}^2}{(2+\beta)^{3/2}(1-4\beta)}\frac{v'^2}{v^3}\,\mathrm{e}^{\phi} - \frac{3}{2}\frac{1+2\beta}{1-4\beta}, \tag{3.18}$$

$$\eta\varepsilon_2 = 3\sqrt{\frac{3}{2}}\frac{m_{Pl}^2}{(2+\beta)^{3/2}}\left[\frac{4-\beta}{1-4\beta}\frac{v'^2}{v^3} - \frac{v''}{v^2}\right]\mathrm{e}^{\phi} - 6\frac{(1+\beta)(1+2\beta)}{1-4\beta}, \tag{3.19}$$

where $e^{\phi} = a^{-2\beta-1}$. From (3.19) we get the second parameter ε_2 expressed by ε_1

$$\varepsilon_2 = \frac{4(4-\beta)}{3}\varepsilon_1 - 3s\frac{v''}{v^2} + (1+2\beta)\left[\frac{2}{3}(7-\beta) + \frac{2(1+2\beta) - 3sv''/v^2}{2\varepsilon_1}\right], \tag{3.20}$$

where $s = \sqrt{3/2}m_{Pl}^2(2+\beta)^{-3/2}\,e^{\phi}$. The inflation takes place if $0 < \varepsilon_1 < 1$. The number of e-foldings, expressed in terms of the tachyon field T and the dilaton field ϕ, is

$$N = \int_T^{T_{\text{end}}} \frac{H}{\dot{T}}\,dT = -\left(\frac{2}{3}\right)^{3/2} \frac{(2+\beta)^{3/2}(1+16\beta+4\beta^2)}{2m_{Pl}^2(1-4\beta)} \int_{T_{\text{end}}}^{T} \frac{v^2}{v'}\,e^{-\phi}\,dT$$
$$+ \frac{1}{3}\int_{T_{\text{end}}}^{T}\left(\frac{11-2\beta}{2(1-4\beta)}\frac{v'}{v} - \frac{v''}{v'}\right)dT. \tag{3.21}$$

Since the dimension of the manifold Y is $8-k$ (see equations (2.1)–(2.3)), the case $\beta = -1/2$ corresponds to the background produced by the D-branes that are warping five-dimensional manifold. In this case, the parameters ε_1 and ε_2 become the standard parameters considered in the tachyon inflation.

4. Conclusions

In this paper, we considered gravity on the non-BPS D3-brane. The action for this system consists of the Einstein–Hilbert action and the DBI-like action for a D3-brane. From this model, we derived the slow-roll parameters. These parameters depend on the potential v, the dilaton field ϕ and the dimension of a manifold on which the background D-branes are warped. For the given potential v and the given background, one can compute these parameters as the functions of T and r. The field r is obtained from the following constraint: $\sigma = 1$ (equation (3.3)) on a D3-brane. The inflation is ended for the fields T and r if $\varepsilon_1(T_{\text{end}}, r_{\text{end}}) = 1$. From ε_1 and ε_2, one can calculate the observable parameters as the functions of T and r. In case when $\beta = -1/2$, we get the well-known parameters for the tachyon inflation.

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